RADIATIVE AND CONVECTIVE HEAT TRANSFER IN A PLANE LAYER OF ABSORBENT CURTAIN

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An examination is made of the thermal state of a plane layer of gray gas injected into a turbulent stream of high temperature gas flowing over a permeable flat plate.

Similarity-type formulations of problems are encountered both in examination of flow near a stagnation point, and also in analysis of the lifting of the boundary layer by intense injection through a porous plate [1]. The examination described relates to the following idealized formulation of the problem (Fig. 1a).

In a plane layer of gray absorbing medium, formed by plane-parallel diffusely radiating surfaces (1—porous plate; 2—boundary of high temperature turbulent gas stream), heat transfer is accomplished by radiation and convection of the layer normal to the surfaces and by molecular heat conduction. All the physical and optical properties of the medium and of the boundary surfaces are assumed to be constant, independent of temperature.

The temperature of the wetted surface of the specimen and also that of the fictitious surface determining the upper bound of the lift-off region, are given.

Also assumed given is the velocity of the injected medium, which is constant throughout the entire lift-off layer. This idealization appreciably facilitates our examination without in principle changing its features.

A very simplified examination of this problem was given in [2]. The special case of a medium with low optical thickness was examined in [3, 4].

The problem was examined in [5] under the assumption that molecular heat conduction in the medium is negligibly small.

In the formulation considered the generalized energy equation has the form

$$\operatorname{div} \mathbf{q}_T + \operatorname{div} \mathbf{q}_k + \operatorname{div} \mathbf{E}_{4\pi} = 0. \tag{1}$$

Here $\mathbf{q}_T$ and $\mathbf{q}_k$ are the vectors of molecular heat conduction and the convective heat flux, and $E_{4\pi}$ is the spherical vector of the radiative heat flux:

$$\operatorname{div} \mathbf{q}_T = -\lambda \nabla^2 T, \quad \operatorname{div} \mathbf{q}_k = \rho c_p (\mathbf{W}, \operatorname{grad} T), \quad \operatorname{div} \mathbf{E}_{4\pi} = -\eta,$$

where η is the resultant volume radiation density.

Taking into account the one-dimensional nature of the heat transfer and also that $-\eta = dE/dy$ in this case, we have

$$\rho_1 c_{p_1} w_1 \frac{dT}{dy} = \lambda \frac{d^2T}{dy^2} - \frac{dE}{dy}, \tag{2}$$

with the boundary condition

$$T = T_1, \ y = 0; \ T = T_2, \ y = \delta. \tag{3}$$

Here E is the resultant hemispheric radiation density. We note that in Eq. (2) w_1 = const inside the lift-off layer. This assertion is well confirmed experimentally. The boundary problem thus formulated is reduced to a complex functional equation for the dimensionless temperature field $\theta = f(h)$ with characteristic parameters N_{B0} and $N_{\lambda\varkappa}$,

$$N_{B0} \frac{d\theta}{dh} = N_{\lambda\varkappa} \frac{d^2\theta}{dh^2} - \frac{1}{4} \frac{d\Phi}{dh}, \tag{4}$$

$$\theta = \theta_1, \quad h = 0; \quad \theta = \theta_2, \quad h = h_0 \quad (h = \varkappa y,\ h_0 = \varkappa\delta),$$
$$\left(\theta = \theta(h) = \frac{T}{T_*},\quad \Phi = \frac{E}{\sigma_0 T_*^4},\ N_{B0} = \frac{c_{p1}\rho_1 w_1}{4\sigma_0 T_*^3},\ N_{\lambda\varkappa} = \frac{\lambda\varkappa}{4\sigma_0 T_*^3},\ \xi = \frac{y}{\delta} = \frac{h}{h_0}\right). \tag{5}$$

Here θ is the dimensionless temperature; Φ is the dimensionless resultant heat flux density; h and h_0 are the optical thickness and the layer thickness, respectively; N_{B0} is a dimensionless parameter describing the radiative-convective ratio in the total heat flux (the Boltzmann parameter); $N_{\lambda\varkappa}$ is a dimensioness parameter describing the radiative-conductive ratio in the total heat flux; T_* is some characteristic temperature; $\varkappa$ is the radiation absorption coefficient; λ is the thermal conductivity of the medium.

The hemispheric resultant radiation density E(h) relative to the layer of gray nonscattering medium examined is determined by the integral relation [6]

$$E(h) = 2A_1(h)E_{0.1} - 2A_2(h)E_{0.2} + 2\int_0^h E_0(\zeta)\Psi(h,\zeta)\,d\zeta - 2\int_h^{h_0} E_0(\xi)\Psi(\zeta,h)\,d\zeta, \tag{6}$$

$$\Psi(h,\zeta) = K_2(h-\zeta) + 2A_1(h)\frac{R_1}{A_1}K_2(\zeta) - 2A_2(h)\frac{R_2}{A_2}K_2(h_0-\zeta), \tag{7}$$

$$\Psi(\zeta,h) = K_2(\zeta-h) - 2A_1(h)\frac{R_1}{A_1}K_2(\zeta) + 2A_2(h)\frac{R_2}{A_2}K_2(h_0-\zeta), \tag{8}$$

$$A_1(h) = \frac{A_1}{D}\left(K_3(h) - 2R_2K_3(h_0)K_3(h_0-h)\right),$$
$$A_2(h) = \frac{A_2}{D}\left(K_3(h_0-h) - 2R_1K_3(h_0)K_3(h)\right) \tag{9}$$
$$\left(K_n(x) = \int_0^1 \exp\frac{-x}{\mu}\,\mu^{n-1}\frac{d\mu}{\mu},\quad D = 1 - 4R_1R_2K_3^2(h_0),\quad E_{0,i} = \sigma_0 T_i^4\right).$$

Here $A_i = 1 - R_i$ is the absorptance (R_i is the reflectance) of the boundary surfaces of the layer of injected gas; and $E_{0,i}$ is the hemispheric equilibrium (blackbody) radiation density; i = 1, 2.

Under actual conditions surface 2, an imaginary interface between some semi-infinite layer of high temperature gas and the injected layer of optically thick gas, is close, as regards its optical properties, to a nonreflecting surface of an absolute blackbody (the turbulent core).

These circumstances will be taken into account below in specific analyses, where it is assumed that $R_2 \equiv 0$, $A_2 = 1$, or $A_2 = \varepsilon_2$, where ε_2 is the emittance of the oncoming gas stream. Nevertheless, for the sake of generality, we retain the radiating system assumed above with gray boundary surfaces. For this reason we take into account later an expression for E(h), represented as the integral relation of Eq. (6).

From term-by-term integration of Eq. (4), and taking into account the stationary nature of the heat transfer processes, we obtain

$$q = 4N_{B0}\theta - 4N_{\lambda\varkappa}\frac{d\theta}{dh} + \Phi(h) \qquad \left(q = \frac{q^\circ}{\sigma_0 T_*^4}\right). \tag{10}$$

Here q is the dimensionless total heat flux.

Integrating Eq. (10) with respect to h from 0 to h, and also taking into account the boundary condition of Eq. (5), we obtain an expression determining the total dimensionless heat flux:

$$q = \frac{4}{h_0}\left\{N_{\lambda\varkappa}(\theta_1 - \theta_2) + N_{B0}\int_0^{h_0}\theta(h)\,dh + \frac{1}{4}\int_0^{h_0}\Phi(h)\,dh\right\}. \tag{11}$$

In computing the second integral we must take into account Eq. (6) and the appropriate rule for integration of the integrand. After a series of transformations Eq. (11) can be written as follows:

$$q = b(h_0) + \frac{2}{h_0}\int_0^{h_0}\left(2N_{B0}\theta(h) + \Psi(h_0 h)\,\theta^4(h)\right)dh \tag{12}$$

$$(b(h_0) = 4h_0^{-1}(N_{\lambda\varkappa}(\theta_1 - \theta_2) + {}^1/_2(A_1(h_0)\theta_1^4 - A_2(h_0)\theta_2^4))).$$

Here

$$\Psi(h_0, h) = K_3(h) - K_3(h_0 - h) + 2A_1(h_0)\frac{R_1}{A_1}K_2(h) - 2A_2(h_0)\frac{R_2}{A_2}K_2(h_0 - h), \tag{13}$$

$$A_1(h_0) = \frac{A_1}{D}\left(\frac{1}{3} - K_4(h_0)\right)(1 - 2R_2K_3(h_0)),$$
$$A_2(h_0) = \frac{A_2}{D}\left(\frac{1}{3} - K_4(h_0)\right)(1 - 2R_1K_3(h_0)). \tag{14}$$

Equation (12) allows us to compute the total heat flux over the surface of the permeable plate wetted by the turbulent stream of high-temperature radiating gas, for the case in which the temperature distribution in the layer of injected gas is known.

Determination of the latter is the main content of the boundary problem of Eqs. (4) and (5), whose solution reduces to the following formal integration of Eq. (10) with respect to h, over the range 0 to h:

$$4N_{B0}\int_0^h \theta(\zeta)\,d\zeta - 4N_{\lambda\varkappa}(\theta(h) - \theta_1) + \int_0^h \Phi(\zeta)\,d\zeta = qh. \tag{15}$$

Using Eq. (12), and carrying out a series of similar transformations, we reduce Eq. (15) to a complex nonlinear integral equation with respect to the dimensionless temperature:

$$\theta(h) = a(h) + \frac{1}{2N_{\lambda\varkappa}}\int_0^{h_0}\left(2N_{B0}\left(\delta - \frac{h}{h_0}\right)\theta(\zeta) + G(h, \zeta)\theta^4(\zeta)\right)d\zeta. \tag{16}$$

Here

$$\delta = 1 \quad (\zeta < h), \qquad \delta = 1 \quad (\zeta > h),$$

$$a(h) = \theta_1 + \frac{\theta_2 - \theta_1}{h_0}h + \frac{1}{2N_{\lambda\varkappa}}\left(\frac{A_1}{D}\theta_1^4\chi_1(h) + \frac{A_2}{D}\theta_2^4\chi_2(h)\right), \tag{17}$$

$$G(h, \zeta) = K_3(\zeta) - K_3|h - \zeta| + \frac{h}{h_0}(K_3(h_0 - \zeta) - K_3(\zeta)) +$$
$$+ 2\frac{R_1}{D}\chi_1(h)K_2(\zeta) + 2\frac{R_2}{D}\chi_2(h)K_2(h_0 - \zeta), \tag{18}$$

$$\chi_1(h) = {}^1/_3 - K_4(h) - h_0^{-1}h({}^1/_3 - K_4(h_0)) + 2R_2K_3(h_0)[K_4(h_0) -$$
$$- K_4(h_0 - h) + h_0^{-1}h({}^1/_3 - K_4(h_0))], \tag{19}$$

$$\chi_2(h) = K_4(h_0) - K_4(h_0 - h) + h_0^{-1}h({}^1/_3 - K_4(h_0)) + 2R_1K_3(h_0)\times$$
$$\times[{}^1/_3 - K_4(h) - h_0^{-1}h({}^1/_3 - K_4(h_0))]. \tag{20}$$

It is quite evident that an analytical examination of the integral equation (16), like the calculation of the corresponding quadratures in Eq. (12), is very difficult. For this reason the whole of the following analysis of the problem is presented with the numerical methods of solution included, as well as certain simplified methods, relevant to determination of the most conservative integral function of the total heat flux.

The integral equation (16) is approximated by a system of nonlinear algebraic equation, which is solved by Newton's numerical method. The corresponding integrals are approximated using the Gauss quadrature formula. Because of the characteristics of the integrand expressions in Eq. (16), appearing when they are represented in discrete form in the boundary region, it is expedient to represent the integral equation (16) in the following form:

$$\theta(h) = \left\{a(h) + \frac{N_{B0}}{N_{\lambda\varkappa}}\int_0^{h_0}\left[\delta(\theta(\zeta) - \theta(h)) - \frac{h}{h_0}\theta(\zeta)\right]d\zeta +\right.$$
$$\left. + \frac{1}{2N_{\lambda\varkappa}}\int_0^{h_0}\theta^4(\zeta)G(h, \zeta)\,d\zeta\right\}\left(1 - h\frac{N_{B0}}{N_{\lambda\varkappa}}\right)^{-1}. \tag{21}$$

Some results of solution of Eq. (21) are shown below, relating to diverse variants of the values of the parameters describing the optical properties of the porous plate R_1 (with $R_2 = 0$), the optical properties of the medium

h_0, the intensity N_{B0} of blowing of adsorbent gas, and also the radiative-convective ratio $N_{\lambda\varkappa}$ in the total heat transfer.

Figures 1 and 2 show dimensionless values of the temperature field in the blocking layer when the surface of the porous plate is absolutely black, $\theta_1 = 0.3$, $\theta_2 = 1.0$, and the radiative-conductive ratio parameter $N_{\lambda\varkappa} = 0.1$ is evidence that the role of molecular heat conduction in the heat transfer is not negligible.

In the case of small values of the parameter N_{B0}, representing weak injection, the nature of the temperature distribution is in many respects reminiscent of the temperature field in a layer of absorbent medium, possessing molecular heat conduction. However, even in this case, the effect of weak blowing in reducing the temperature level of the blocking layer is noticeable.

With increase of the optical thickness of the blocking layer we note some increase of the temperature in the main part of the layer adjacent to the cooled surface, and also a corresponding decrease in the layer adjacent to the hot surface (Fig. 1b) in comparison with the distribution established in a layer of weakly absorbent gas.

Here, with increase of injection, the picture changes, the difference being that the region adjacent to the hot gas is so much developed that the temperature of the optically thick gas (Fig. 1b) is lower than that in the layer of weakly absorbing gas over the whole blocking layer. A distinctive luminescence of the gas is produced in the cold layer region. This becomes particularly noticeable in the gas layer with optical thickness $h_0 \approx 1.0$, when the interaction effects in the medium are greatest.

For the case in which the surface of the porous plate has a high reflectance the general picture of temperature distribution is retained, but the temperature level is increased, due to boundary reflection effects. Here a particular rise of temperature is noticeable in the region adjacent to the reflecting surface (Fig. 2). Thus, representation of the porous plate with a high surface reflectance, besides the effect of direct thermal protection from radiation, is accompanied by a certain increase in temperature of the optically thick gas blowing through the layer.

Figure 3 shows dimensionless total heat flux as a function of the optical thickness of the layer, for various values of N_{B0}. As we would expect, with increase of N_{B0}, which describes the intensity of blowing, the total dimensionless heat flux changes sign for certain values of h_0 and N_{B0}. In other words, the surface of the porous plate changes from a heat-receiving to a heat-liberating surface. For certain conditions the resultant heat transfer to the layer is identically zero ($q = 0$). Figure 4 shows the relation $N_{B0} = f(h_0)$, describing the adsorbent-gas injection rate required to maintain the indicated thermal insulation state, as a function of the optical thickness of the gas.

There is a sharply pronounced dependence of N_{B0} on h_0 in the region of relatively weakly absorbing gas, when the insignificant amount of absorbing component in the gas permits a very noticeable increase in the flow rate of injected gas. A highly reflecting surface (Fig. 4), when the optical density h of the blown gas is less than 1.0, is an effective arrangement.

In practice we are interested in simplified treatments which allow approximate analysis of heat transfer in the region of the line of boundary layer lift-off from the surface of the permeable plate. The above approach is based on determination of the total resultant heat flux q, which, being an integral function of the temperature distribution, is a conservative concept, so that, in determining it, we use an approximate, but quite reasonable, temperature distribution.

Strictly speaking, the solution of Eq. (12) determines the total heat flux q over the porous plate surface. Being constant in any section of the layer, the flux q, composed of conductive, convective, and radiative components, is redistributed in complete accordance with the temperature distribution and the role of the various kinds of heat transfer. Attention should be drawn to the additive form of Eq. (12) for q, where, in contrast with the solution for calculating local q, there is a term representing molecular heat conduction in free form. This somewhat reduces the error associated with bringing in the approximate temperature distributions.

Using a linear distribution for the equilibrium radiation:

$$\theta^4(h) = \theta_1^4 + (\theta_2^4 - \theta_1^4)\, h/h_0 \tag{22}$$

in determining the radiative component of the total heat flux, the expression for q can be written as follows:

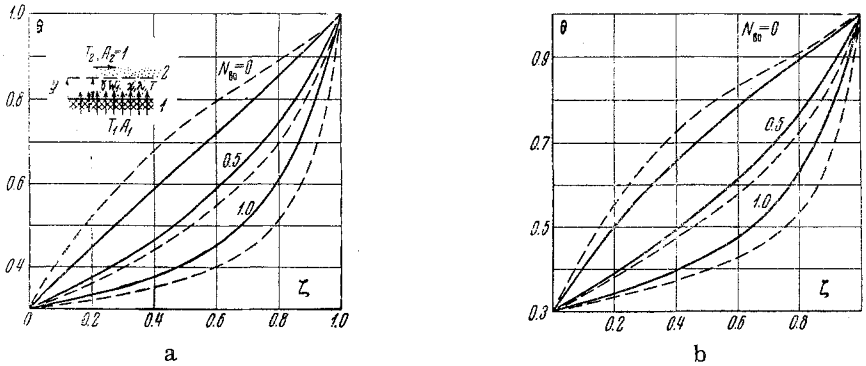

Fig. 1. Dimensionless temperature distribution in the blocking layer for various values of N_{B0} with $\theta_1 = 0.3$, $\theta_2 = 1.0$, $N_{\lambda\varkappa} = 0.1$, $A_1 = A_2 = 1.0$ and with optical thickness a) $h_0 = 0.5$ (solid lines), $h_0 = 1.0$ (dashed lines); b) $h_0 = 2.0$ (solid lines), $h_0 = 3.0$ (dashed lines).

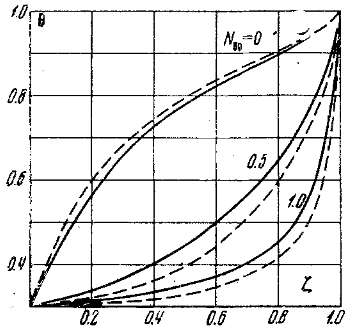

Fig. 2. Dimensionless temperature distribution in the blocking layer with $\theta_1 = 0.3$, $\theta_2 = 1.0$, $N_{\lambda\varkappa} = 0.1$, $A_1 = 0.2$, $A_2 = 1.0$ and optical thickness $h_0 = 0.5$ (solid lines) and $h_0 = 1.0$ (dashed lines), for various values of N_{B0}.

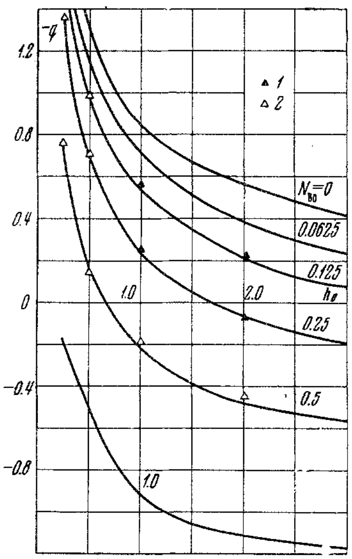

Fig. 3. Dimensionless values of the total heat flux q as a function of the optical thickness of the layer, for various values of N_{B0} ($\theta_1 = 0.3$, $\theta_2 = 1.0$, $N_{\lambda\varkappa} = 0.1$, $A_1 = A_2 = 1.0$); points 1 were calculated with Eq. (27), and points 2 by Eq. (28).

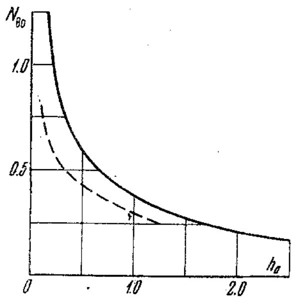

Fig. 4. N_{B0} as a function of the optical thickness h_0 of the layer, for $q = 0$, $\theta_1 = 0.3$, $\theta_2 = 1.0$, $N_{\lambda\varkappa} = 0.1$; $A_1 = A_2 = 1.0$ (solid line) and $A_1 = 0.2$, $A_2 = 1.0$ (dashed line), respectively.

$$q = -\frac{4N_{\lambda\varkappa}}{h_0}(\theta_2 - \theta_1) + \frac{4N_{B0}}{h_0}\int_0^{h_0}\theta(h)\,dh - A_{12}(h_0)(\theta_2^4 - \theta_1^4). \quad (23)$$

Here $A_{12}(h_0)$ is the resolvent optical and geometrical functional, which for gray boundary surfaces in general, has the form

$$A_{12}(h_0) = 2h_0^{-1}(2/3 - h_0^{-1}(1/2 - 2K_5(h_0)) + 2(1/3 - K_4(h_0))(A_1(h_0)R_1 + A_2(h_0)R_2))). \quad (24)$$

Determination of the convective flux component is decided in many respects by the approximation for the temperature distribution in the layer. It turns out to be of little use to take the relation

$$\theta(h) = \theta_1 + (\theta_2 - \theta_1)\frac{e^{Nh} - 1}{e^{Nh_0} - 1} \qquad \left(N = \frac{N_{B0}}{N_{\lambda\varkappa}}\right) \quad (25)$$

as an approximate solution of Eq. (21) in a diathermal medium. It is more efficient to construct approximate solutions valid in specific ranges of N_{B0}.

For $N_{B0} \leq 0.25$, when the dynamic effects of convective transfer are of the same order as the radiative and conductive effects ($N \sim 1.0$), the temperature distribution in the layer is determined to a considerable degree by the radiative heat transfer, which is strongly coupled to the medium. In this case, in determining the convective component of q in Eq. (23) we can use an approximate temperature distribution, determined as the arithmetic mean of the distributions in a fixed, intensely absorbing medium

$$\theta(h) \approx \theta_1 + (\theta_2 - \theta_1)\frac{1 - K_2(h)}{1 - K_2(h_0)}, \quad (26)$$

and in a moving, weakly absorbing medium, Eq. (25).

The formula then obtained, representable in the following form:

$$q = 4N_{B0}\left\{\theta_1 - \frac{1}{2}\left[(e^{Nh_0} - 1)^{-1} + \frac{1}{h_0}\left(\frac{1}{N} - \frac{h_0 - (1/2 - K_3(h_0))}{1 - K_2(h_0)}\right)\right](\theta_2 - \theta_1)\right\} - A_{12}(h_0)(\theta_2^4 - \theta_1^4) \quad (27)$$

is valid for $h_0 \geq 1.0$ and $N_{B0} \leq 0.4$.

For $N_{B0} \geq 0.4$ and $h_0 \leq 1.0$ it is better to use an approximate expression whose convective component is determined in the approximation for the temperature distribution, which is taken as the arithmetic mean of the distributions of fixed and moving, weakly absorbing media:

$$q = 4N_{B0}\left\{\frac{1}{4}(\theta_2 + 3\theta_1) - \frac{1}{2}\left(\frac{1}{Nh_0} + (e^{Nh_0} - 1)^{-1}\right)(\theta_2 - \theta_1)\right\} - A_{12}(h_0)(\theta_2^4 - \theta_1^4). \quad (28)$$

The results of calculations according to Eqs. (27) and (28) are in good agreement with results of numerical solution of Eq. (12) (the error does not exceed 5%). In optically thin media the level of agreement of calculations according to Eq. (28) increases (Fig. 3).

The similarity type of approximate solution, although trivial in concept is rather complex because the three components of the resultant heat transfer are considered simultaneously.

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